A Generative Adversarial Framework for Bounding Confounded Causal Effects

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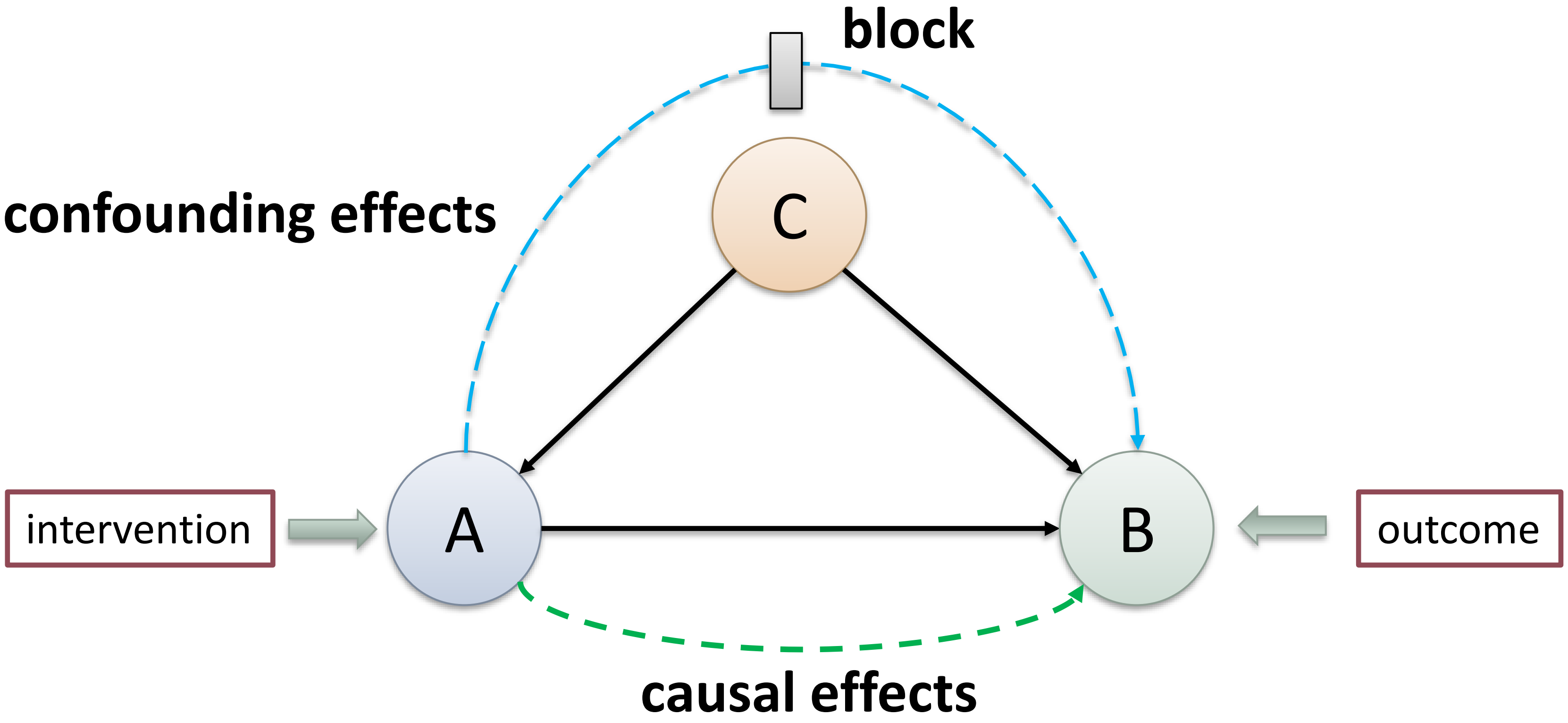
Causal Inference

Economics **Healthcare** **Fair Machine Learning**

Inferring causal effects from observational data is an important task in many fields.

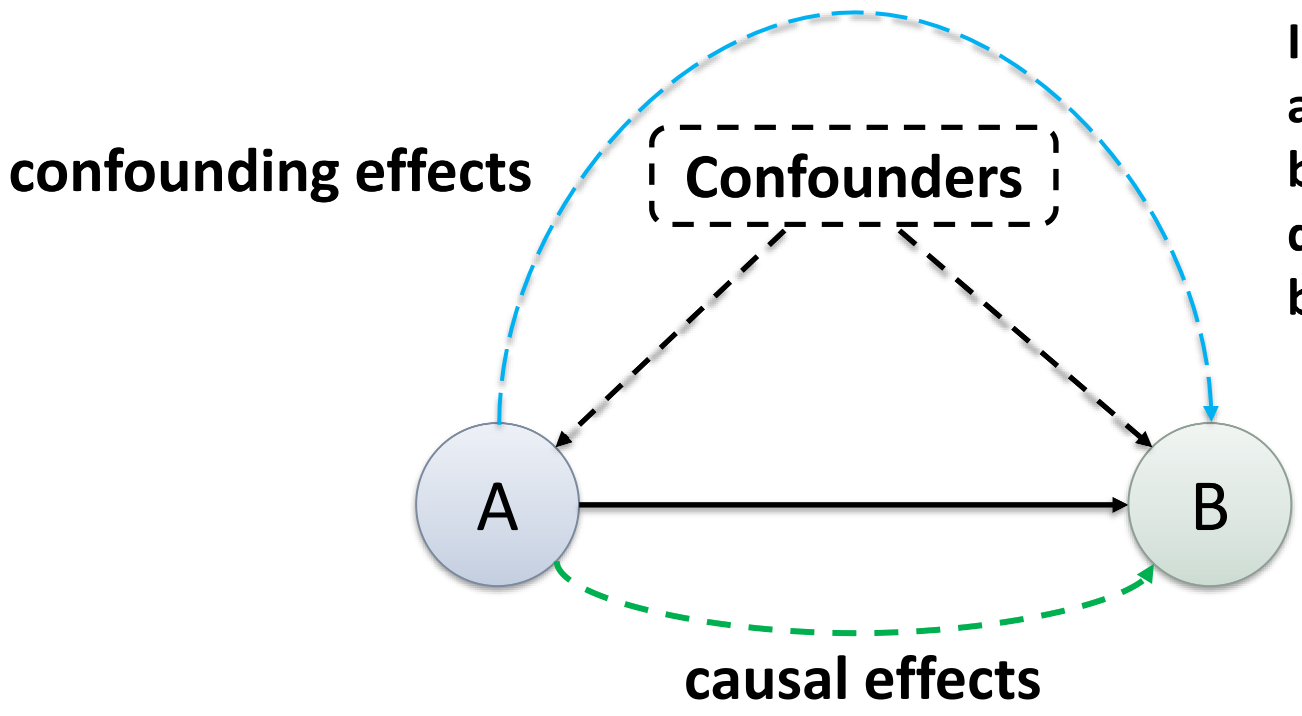
Pearl's structural causal model is a decent and widely adopted framework for conducting causal inference.

Inferring Causal Effects



Under the assumption of no hidden confounding, the ACE can be calculated using the well-known back-door criterion.

Inferring Causal Effects



In an unidentifiable situation, any estimation of ACEs only based on the observational distribution is not guaranteed to be correct.

When hidden confounders exist, the ACE may not be uniquely calculated from the observational data without further assumptions, known as the unidentifiable problem.

Unidentifiable Problem

- **Causal graph**

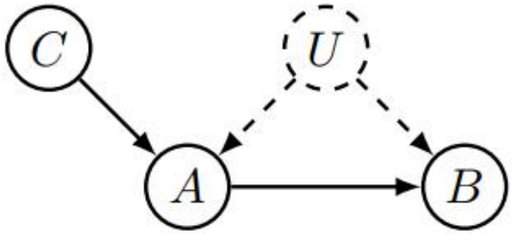


Figure 1: Example: A, B, C are observed variables and U is a hidden variable.

- **Two different causal models**

Table 1: Equation $f_A(c, u)$ for determining values of A.

U	C	$A = f_A$
0	0	0
0	1	0
1	0	1
1	1	1

Table 2: Equations $f_B^1(a, u)$ and $f_B^2(a, u)$ for determining values of B.

U	A	$B = f_B^1$	$B = f_B^2$
0	0	0	0
0	1	1	0
1	0	0	0
1	1	1	1

Unidentifiable Problem

The two models completely agree with $P(A, B, C)$, but differ in the ACE of A on B.

Previous Work on Bounding ACE

- [Balke and Pearl, 1997] developed a constrained optimization problem for discovering bounds from the observational data.
- The general idea is to shift the randomness of the causal model from the distributions of $\boldsymbol{U}$ to the distributions of mappings, and then use linear programming to search for distributions that lead to lower or upper bound of the ACE.

- Limitations: limited to categorical endogenous variables and cannot directly extend to the continuous domain.

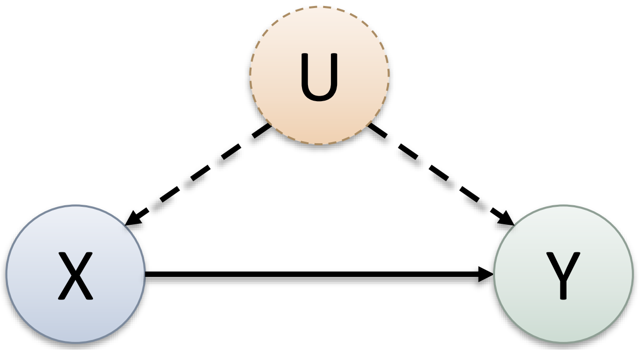
Our Work

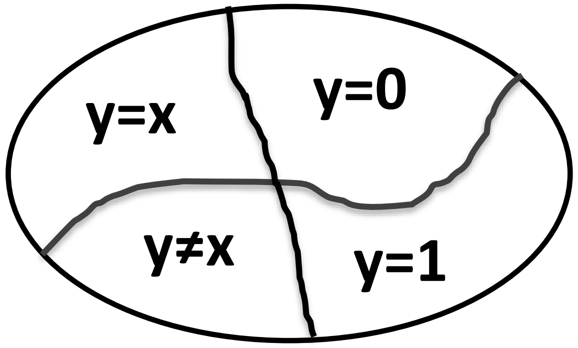
- We extend the previous method for bounding ACEs to continuous and possibly high dimensional variables.
- We propose to parameterize the unknown exogenous random variables and structural equations of a causal model using neural networks and implicit generative models.
  - Estimate response functions from $\mathrm{PA_V}$ to V by neural networks with a certain network structure.
  - Use the implicit generative model to generate the distribution for the response-function variable.
  - Parameterize the causal model by expressing it with response-function variables.
  - Formulate an adversarial learning problem for computing the bounds of the ACE.

Response-function

- **Response-function**

  To partition the domain of each exogenous variable into a limited number of equivalent classes, each inducing a distinct functional mapping between endogenous variables. These functional mappings are called the ***response functions***.



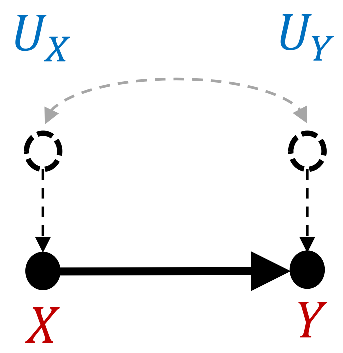


Domain of U

Response-function Variables

- Response-function variables $\boldsymbol{r}$ are used to parameterize the causal model.
  - Categorize the unknown domain of $\boldsymbol{U}$ into limited number of equivalent regions, each of which is denoted by a value of ***a response-function variable***.
  - As a result, all uncertainties in the causal model parameterized by $P(\boldsymbol{r})$.
  - Search bounds by a linear programming problem with $P(\boldsymbol{r})$ as variables.

- **Example**



$$r_X = \begin{cases} 0 & \text{if } f_X(u_X) = x_0 \\ 1 & \text{if } f_X(u_X) = x_1 \end{cases}$$

$$r_Y = \begin{cases} 0 & \text{if } f_Y(x_0, u_Y) = y_0,\ f_Y(x_1, u_Y) = y_0 \\ 1 & \text{if } f_Y(x_0, u_Y) = y_0,\ f_Y(x_1, u_Y) = y_1 \\ 2 & \text{if } f_Y(x_0, u_Y) = y_1,\ f_Y(x_1, u_Y) = y_0 \\ 3 & \text{if } f_Y(x_0, u_Y) = y_1,\ f_Y(x_1, u_Y) = y_1 \end{cases}$$

Coping with Continues Domain

- For each endogenous variable V, a neural network $v = h_V(paV; \theta_V)$ is used as a universal estimator of response functions from PA_V to V.
  - Input pa_V and parameters $\theta_{\mathrm{V}} \in \Theta_{\mathrm{V}}$.
  - We treat Θ_{V} as the response variable.
  - If $PA_V = \emptyset$, directly use $v = \theta_V$ to represent a trivial mapping.

- To generate different distributions for θ_{V}, we adopt the implicit generative model, which generates data by transforming some random noise to the data via some deterministic function.
  - The random noise $\boldsymbol{z}_V$ is taken as input and transformed into θ_V via a neural network $G_V(\boldsymbol{z}_V)$, referred to as a generator.

Parameterizing Causal Models

- Obtain an implicit generative model for each $V \in \mathbf{V}$:

$$v = h_V(pa_V; G_V(\boldsymbol{z}_V)) \longrightarrow U_V$$

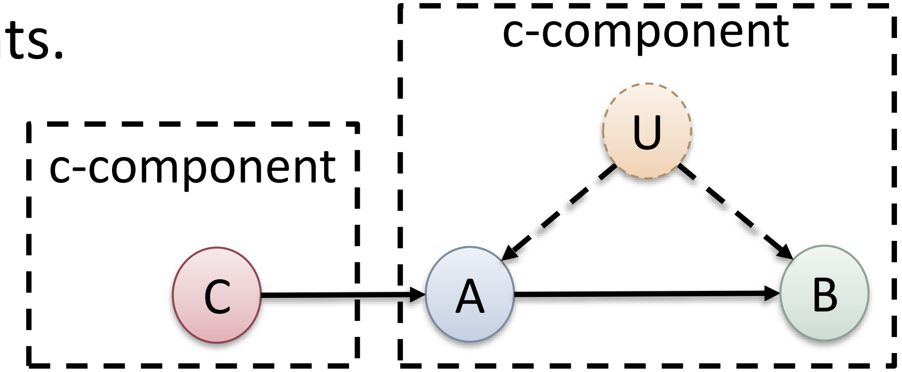
Definition 3. *For a causal model* $\forall V \in \boldsymbol{V}$, $v = f_V(pa_V, u_V)$, *its parameterized version is given by*

$$\forall V \in \boldsymbol{V}, v = h_V(pa_V; G_V(\boldsymbol{z}_V))$$

where generators $G_V(\boldsymbol{z}_V)$ *contain parameters that are to be learned from data.*

Encoding Independence Assumptions

- Since Θ_V is a representative of U_V, it should inherit the independence relationship between U_V and other exogenous variables.
  - Θ_{V1} and Θ_{V2} should be (in)dependent if U_{V1} and U_{V2} are known to be (in)dependent.
  - Use same random noise for generators G_{V_1} and G_{V_2} if U_{V_1} and U_{V_2} are dependent.
- Any causal graph can be decomposed into a number of disjoint components, called ***c-components***, such that any pair of exogenous variables are correlated if they belong to the same component and independent if they belong to different components.



Bounding ACEs

- The ACE of A on B is given by $E[B|do(a_1)] - E[B|do(a_2)]$.
- For any intervention $do(a')$, we directly perform it to modify the parameterized causal model as:

$$\boxed{a = a'}; \forall V \neq A, v = h_V(\mathrm{pa}_V; G_V(\boldsymbol{z}_V))$$

- We estimate the value of an ACE of A on B by sampling B from the intervened parameterized causal model.
  - Denoted by $ACE(G;\ a_1, a_0)$
- Compute lower bound of ACE by minimizing $ACE(G;\ a_1, a_0)$.

Bounding ACEs

- We want the causal models searched in learning process to be confined to those agree with a given observational distribution $P(\boldsymbol{v})$.
- The generative adversarial learning framework is adopted to ensure that generated distribution is close to the observational distribution.
- A discriminator is trained to minimize the discrepancy between the generated distribution and the observational distribution.
- The objective function is given by $\max_{D} V(G, D)$ where

$$V\ (G;\ D) = E_{\boldsymbol{v}\ \sim\ P(\boldsymbol{v})}[\log D(\boldsymbol{v})] + E_{\boldsymbol{z}\ \sim\ P(\boldsymbol{Z})}\ [\log(1 - D(G(\boldsymbol{z})))]$$

Bounding ACEs

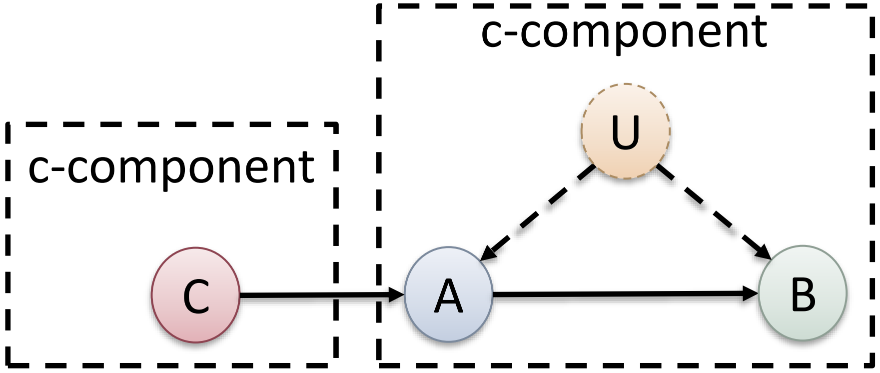
- Combining two partial objectives, to obtain the lower bound (similarly for the upper bound), we would like to learn generators G that minimize $ACE(G;\ a_1, a_0)$ subject to that $\max_D V(G, D) \leq m + \eta$.
  - m is the theoretical minimal value of $\max_D V(G, D)$.
  - η is a threshold which specifies how close we want the generated distribution to the observational distribution.

Problem 1. *Given a causal graph and the data, the lower bound (similarly for the upper bound) of the ACE of A on B is computed by solving the optimization*

$$\min_G \max_{\lambda \geq 0} \max_D \{ACE(G; a_1, a_0) + \lambda(V(G, D) - m - \eta)\}$$

Example

- Causal graph and equations



$$c = f_C(\mathbf{u}_C) \qquad c = G_C(\mathbf{z}_1)$$
$$a = f_A(c, \mathbf{u}_A) \quad \xRightarrow{\text{parameterized}} \quad a = h_V(c; G_A(\mathbf{z}_2))$$
$$b = f_B(a, \mathbf{u}_B) \qquad b = h_V(a; G_B(\mathbf{z}_2))$$

- Architecture of neural networks

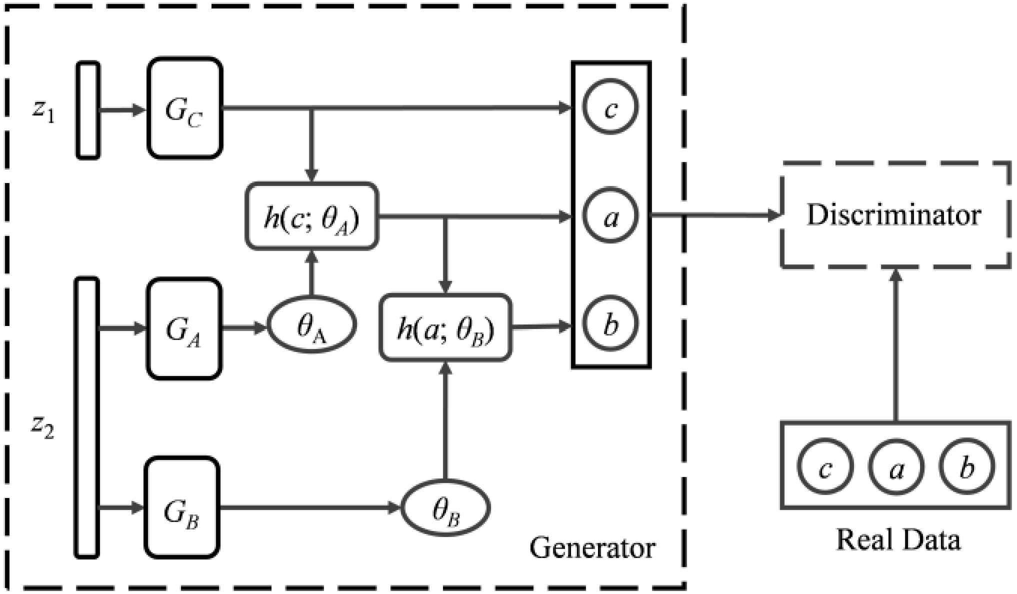


Figure 2: The network architecture with three generators G_A, G_B, G_C for the causal graph shown in Figure 1. A discriminator is used to measure the difference between the generated data and the real data.

Linear Causal Models: A Special Case

- Linear causal model assume that all structural equations in the model are linear.
- For each variable V, we define the response function as the inner product between a parameter vector and the input, i.e., $v = GV(\boldsymbol{z}_V) \cdot [\mathrm{pa}_V, 1]^T$.

Proposition 1: *Let C be an instrumental variable for ACE of A on B, then both bounds will converge to $\frac{cov(B,C)}{cov(A,C)}(a_1 - a_0)$ if the generated distribution converges to the observational distribution.*

Experiments

- **Experiments are conducted on synthetic and real-world data.**

- **Baselines**:
  - **Linear/logistic regression**: We build a linear/logistic regression on the outcome using all observed variables, and then compute the ACE based on the coefficient of the treatment variable.
  - **Instrumental variable** estimation: We implement this method following the classic instrumental variable formula.
  - **Propensity score** adjusted regression: We adopt the propensity score adjusted regression and follow other method to handle continuous variables.

Experiments

- **Synthetic Data:**

ACE of X on Y

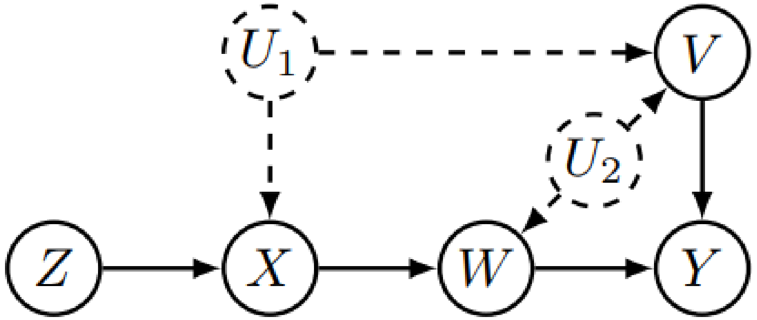


Figure 3: The causal graph of synthetic data: Z, X, W, V, Y are observed variables and U_1, U_2 are hidden variables.

- **Structural Equations:**

$$u_1 = \epsilon_1, \quad u_2 = \epsilon_2, \quad z = \text{Uniform}(\theta_1^z, \theta_2^z) + \epsilon_z$$
$$x = \theta_0^x + \theta_1^x z + \theta_2^x u_1 + \epsilon_x, \; w = \theta_0^w + \theta_1^w x^2 + \theta_2^w u_2 + \epsilon_w$$
$$v = \theta_0^v + \theta_1^v u_1 + \theta_2^v u_2 + \epsilon_v, \; y = \theta_0^y + \theta_1^y w + \theta_2^y v + \epsilon_y$$

Experiments

- **Results of Synthetic Data:**

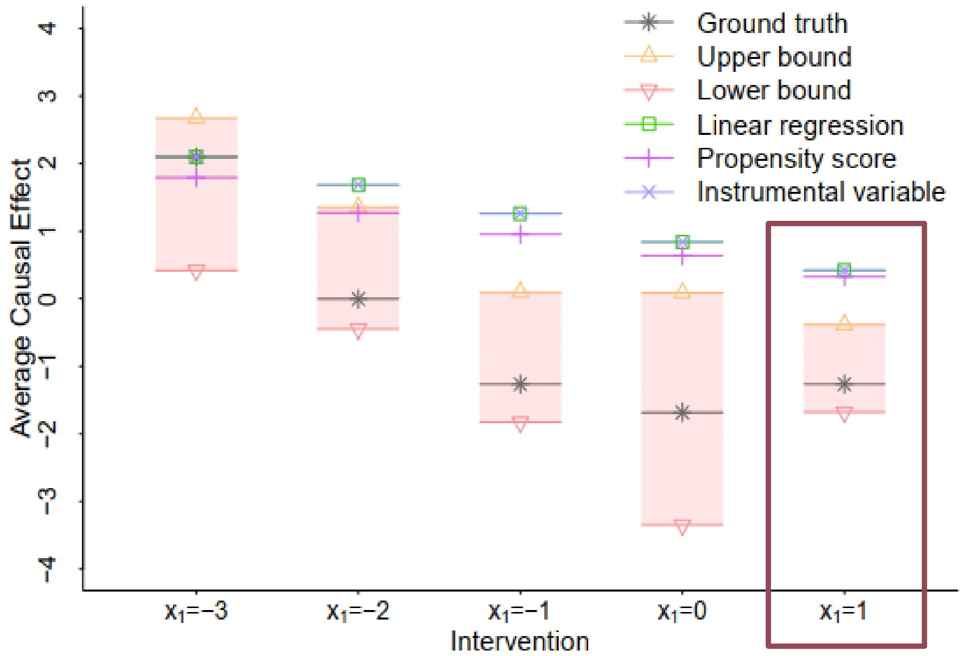


Figure 4: Average causal effects with different interventions ($x_0 = 2$) on the nonlinear synthetic dataset.

- Our upper bound and lower bound cover the ground truth in all interventions.
- Other baseline methods cannot produce accurate estimations and fall outside the bounds in most cases.

Experiments

- **Adult Data:**

ACE of *edu_level* on *income*

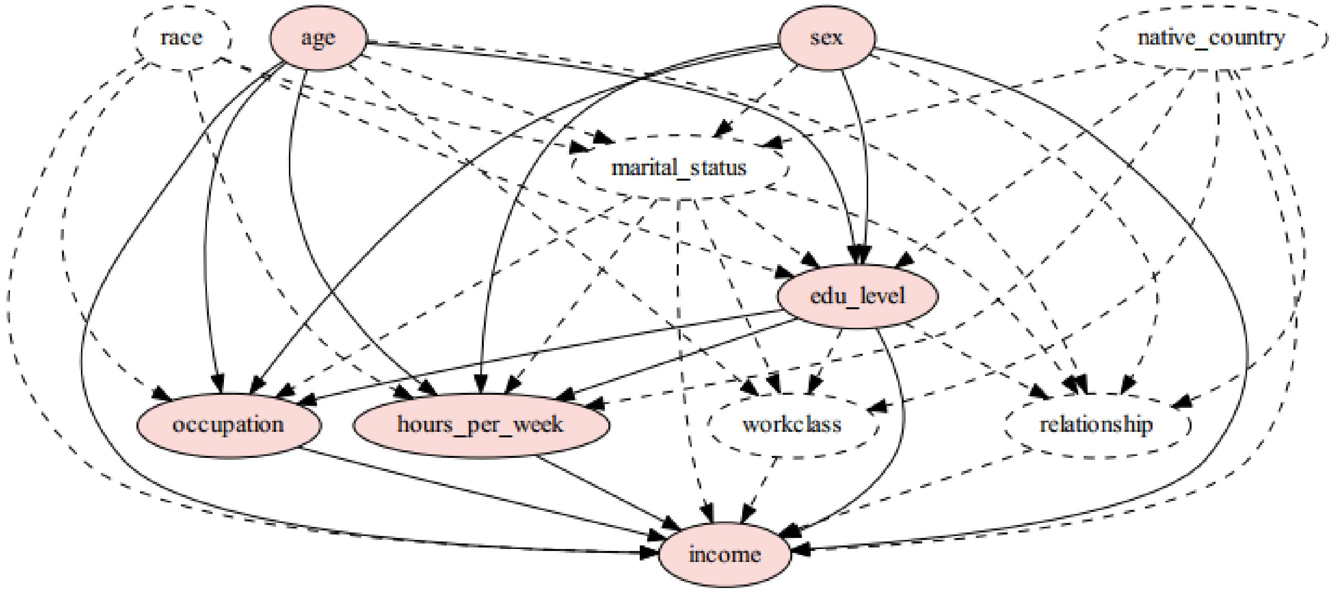


Figure 5: The causal graph of the Adult dataset.

Tool: TETRAD for building the causal graph (using the classic PC algorithm)

Experiments

- **Results of Adult Data:**

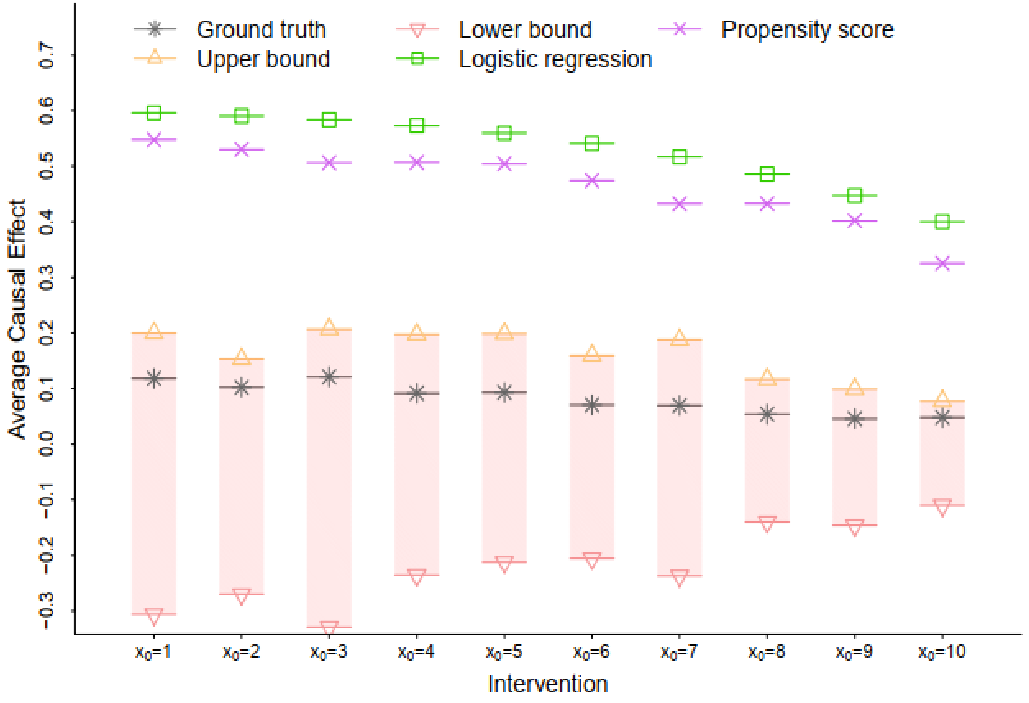


Figure 6: Average causal effects with different interventions ($x_1 = 16$) on the Adult dataset.

- The ground truth falls in the range of the upper bound and lower bound in all interventions.

- Other baseline methods including the logistic regression and propensity score cannot produce accurate estimations and fall outside the bounds in all cases.

Conclusions

- Proposed a bounding method for estimating average causal effects (ACEs) from observational data with hidden confounding.
- Parameterized the causal model using implicit generative models and builds an adversarial network to formulate a constrained optimization problem for computing the bounds.
- Showed that encoding the linear assumption can make the bounds converge to a fixed value.
- Conducted experiments on both synthetic and real-world datasets.

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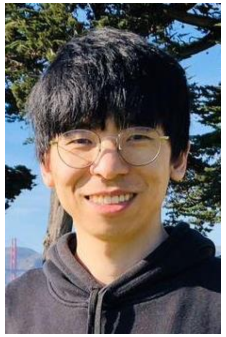


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